# Amplitude Distribution Independent Prediction of Fatique Life

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*It is shown that the fatigue life of vibrating machine parts may be predicted by spectro-analysis without information on amplitude distributions. For this purpose a square mean, instead of an arithmetic mean in the shape of the Palmgren/Miner's rule, is proposed as the cumulative damage hypothesis.*
*The spectro-analysis is performed on a newly introduced damage gradient function corresponding to the load time function. This damage gradient function may be analyzed by discrete (fast) Fourier transformation.*
*The article contains some examples of application including random processes.*

*Es wird gezeigt, daß die Lebensdauer von schwingenden Maschinenbauteilen durch Spektralanalyse ohne Informationen über Amplitudenverteilungen berechnet werden kann. Zu diesem Zweck wird ein quadratischer Mittelwert als kumulative Schädigungshypothese vorgeschlagen anstelle eines arithmetischen Mittelwertes in Form der Palmgren/Miner-Regel. Die Spektralanalyse mittels schneller Fouriertransformation erfolgt an einer neu eingeführten Schädigungsgradientenfunktion, die der Beanspruchungs-Zeit-Funktion zugeordnet ist.*

Betriebsfestigkeit, Lebensdauerberechnung, Spektralanalyse, verteilungsfrei

сопротивление усталости, расчет долговечности, спектральный анализ, без функции распределения

structural fatigue, fatigue life estimation, spectro-analysis, distribution independent

résistance de fatique, évalution durabilité, analyse spectrographique, indépendant de distribution

## 1. Introduction

Three main input values are necessary for fatigue life prediction of vibrating machine parts: firstly information on fatigue behaviour of material, secondly a cumulative fatigue damage hypothesis, and thirdly an effective description of the load time function. The typical load has to be outlined and so recorded that the essential damage influences are contained. The cumulative frequency distribution of amplitudes is proved to be most responsible for fatigue life. Therefore, at present it is customary to calculate the fatigue strength on the basis of amplitude distribution. These distributions may be detected by means of different counting methods, which generally provide different results, because only selected points are processed, e.g. maxima and minima. However, when the digital counting result is obtained, then the fatigue life can be directly calculated by a cumulative sum formula- without any spectrum. Thus, so far the spectro-analysis on fatigue strength has only been used for additional process information. Such analysis is based on samples and hence requires processing of all values instead of selected points. Besides, there are already some formulas for fatigue life prediction on the basis of samples, but until now they have always needed knowledge about the special amplitude distribution. Independent of these problems, in recent years the spectro-analysis instrumentation has been developing rapidly.
In the following derivation is shown, which contributions have to be realized for a sole application of spectro-analysis on fatigue strength — without consideration of amplitude distributions.

## 2. Fatique life due to a stationary Gaussian load process

So far the Gaussian process has been investigated and applied most of all. Miles [1] was the first to propose a closed solution for spectro-analytical calculation of fatigue life:

$$\sigma_r = \Gamma\left(\frac{\varphi}{2}+1\right)^{\frac{1}{\varphi}} \cdot \sqrt{2} \cdot \sigma_{\text{eff}} \tag{1}$$

where

$\Gamma$: Gamma function. $\varphi$: exponent of Woehler's curve ($s$-$N$-curve, i.e. stress amplitude against Number of cycles to failure)

$\sigma_{\text{eff}}$: root mean square of all samples of the stress time function $\sigma(t)$. Only $\sigma_{\text{eff}}$ is calculable by spectro-analysis

$\sigma_r$: reduced stress, by which according to Miles, the fatigue life $N_{Mi}$ occurs:

$$N_{Mi} = K_w \cdot \sigma_r^{-\varphi} \tag{2}$$

where $K_w$: constant of $s$-$N$-curve

Eq. (2) means a straight $s$-$N$-curve in a double logarithmic diagram. From Eqs. (1) and (2) the fatigue life by Miles is as follows:

$$N_{Mi} = \frac{K_w}{\Gamma\left(\frac{\varphi}{2}+1\right) \cdot 2^{\frac{\varphi}{2}} \cdot \sigma_{\text{eff}}^{\varphi}} \tag{3}$$

The foundation of this equation is the assumption of linear damage, i.e. the formula by Palmgren [2] and Miner [3] without endurance limit (Corten/Dolan [4] linearified):

$$N_M = \frac{\sum_{K=1}^{m} n_K}{\sum_{K=1}^{m} \frac{n_K}{N_K}} \tag{4}$$

where

$N_M$: fatigue life by Palmgren/Miner

$m$: number of amplitude block steps of a sine function with various amplitude values $\sigma_K$

$k$: block step

$n_K$: number of load cycles per block step

$N_K$: number of load cycles that will produce fatigue failure due to $\sigma_K$

Miles replaced the sums in Eq. (4) by integrals of frequency density $p_1(\hat{\sigma})$ of maxima $\hat{\sigma}$, for example:

$$\sum_{K=1}^{m} n_K = \int_0^{\infty} p_1(\hat{\sigma})\,\mathrm{d}\hat{\sigma} = n_1 \int_0^{\infty} f_1(\hat{\sigma})\,\mathrm{d}\hat{\sigma} \tag{5}$$

because of $p_1(\hat{\sigma}) = n_1 \; f_1(\hat{\sigma})$

where $n_1$: total number of maxima

$f_1(\hat{\sigma})$: one-dimensional density function of maxima

The maxima of a stationary ergodic Gaussian process have a Rice distribution [5, 6]. By contrast, the samples have a Gaussian distribution.

Particulary in a narrow-band Gaussian process the Rice probability density of maxima goes over to a Rayleigh density:

$$f_1(\hat{\sigma}) = \frac{\hat{\sigma}}{\sigma_{\text{eff}}^2} \cdot \exp\left[-\frac{\hat{\sigma}^2}{2\sigma_{\text{eff}}^2}\right] \tag{6}$$

Thus, the formula by Miles (4) can be described in consideration of Eqs. (5) and (6) as a ratio of integrals of the Rayleigh density (6). Miles essentially simplified this ratio by substituting the integrals through the complete Gamma function:

$$\Gamma(z+1) = 2^{-z} \int_0^\infty x^{2z-1} \cdot \exp\left[-\frac{1}{2}x^2\right] dx \tag{7}$$

Eq. (4) only becomes Eq. (3) by means of Eq. (7). Consequently, Eq. (3) is only valid for a *stationary ergodic narrow-band* Gaussian load process, since Eq. (3) is based on the Rayleigh distribution of maxima. Spectro-analytical formulas proposed later may often be reduced to Miles'formula.

### 3. Distribution independent calculation of fatique life

Of course, it is conceivable to apply other kinds of distribution instead of distribution (6). All spectro-analytical formulas, which are based on the Palmgren/Miner rule (4) may be reduced to the following fundamental type:

$$N_M = \frac{1}{L\{f_1(\hat{\sigma})\} \cdot \sigma_{\text{eff}}^{\varphi}} \tag{8}$$

where $L$: functional expression dependent on the one-dimensional probability density $f_1$ of maxima $\hat{\sigma}$.

The root mean square $\sigma_{\text{eff}}$ can be obtained by the total power $S_{\sigma\text{ges}}$ of the process $\sigma(t)$:

$$\sigma_{\text{eff}}^2 = S_{\sigma\text{ges}} = \int_{-\infty}^{\infty} \tilde{S}(\omega)\, d\omega \tag{9}$$

where $\tilde{S}$: power spectral density and $\omega$: angular frequency. Eq. (8) is based on the assumption of linear cumulative damage. Only this kind of damage supplies a result independent of the sequence of load steps. Moreover, linear damage according to Eq. (4) represents an arithmetic mean of all additional damage $d\,D_K$ of the steps $k = 1, 2, \ldots. m$:

$$d\,D_K = n_K \cdot \frac{1}{N_K} \tag{10}$$

The reciprocal cycles $1/N_K$ are here designated as „Step Damage Gradient $\Delta D_K$". Considering $\Delta D_K$, the Palmgren/Miner rule (4) without endurance limit is an arithmetic mean of step damage gradients $\overline{\Delta D_K}$:

$$N_M = \frac{1}{\overline{\Delta D_K}} \tag{11}$$

where

$$\overline{\Delta D_K} = \frac{1}{\sum_{K=1}^{m} n_K} \cdot \sum_{K=1}^{m} n_K \cdot \Delta D_K \tag{12}$$

When Eq. (11) is compared to Eq. (8), then it is clear that the arithmetic mean of amplitudes cannot be sufficiently described by the root mean square of samples, because there is not any relationship between both values.
Incidentally, that is the reason why beside the root mean square $\sigma_{\text{eff}}$ it is always necessary to use additionally a functional expression $L$ for charakterization of the amplitude distribution — if the Palmgren/Miner rule is utilized. One has to choose another mean to avoid the knowledge about amplitude distribution. For this purpose the root mean square of amplitudes is suitable. For example, a zero mean sine function $x(t)$ of a constant frequency without change of mean stress and with various amplitudes $x$, see Fig. 1. provides:

$$\overline{\overline{x_K{}^2}} = \sqrt{2} \cdot x_{\text{eff}} \tag{13}$$

where

$x_{\text{eff}}$: root mean square of samples $x(t)$

$\overline{\overline{x_K{}^2}}$: root mean square of amplitudes $x_K$:

$$\overline{\overline{x_K{}^2}} = \sqrt{\overline{x_K{}^2}} = \frac{1}{\sum\limits_{K=1}^{m} n_K} \cdot \sum_{K=1}^{m} n_K \cdot x_K{}^2 \tag{14}$$

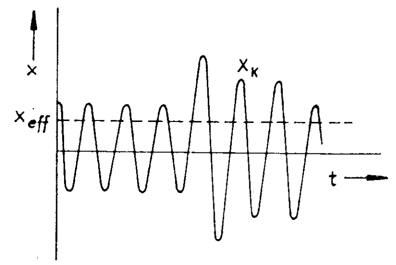


Fig. 1 Sine function with various amplitudes $x_K$

Hence there exists between both root mean square values a simple relation, which is independent of distribution of amplitudes. Therefore it was proposed [7, 8] to use, instead of Eq. (11), the root mean square $\overline{\overline{\Delta D_K{}^2}}$:

$$N_L = \frac{1}{\overline{\overline{\Delta D_K{}^2}}} \tag{15}$$

However, in this case the mean square has to be constructed by a root mean square of sampled damage gradients $\Delta D(t)$. These gradients follow from the sampled stress time function $\sigma(t)$, i.e. everey sample $\sigma$ of time $t$ provides one sample $\Delta D$.
In this manner a „Damage Gradient Function $\Delta D(t)$" is produced. The function $\Delta D(t)$ is detected by the $s$-$N$-curve, e.g.

$$\Delta D(t) = \frac{1}{K_W} \cdot \sigma^q(t) \tag{16}$$

here for $N(\sigma)$ as straight line in a log $\sigma$-log $N$-diagram. The damage capacity is only assumed for samples $\sigma(t)$ above the mean value, because in a zero mean load time function, an equally large negative sample belongs to every positive sample. This point of view corresponds to the structure of an $s$-$N$-curve.

The root mean square $\Delta D_{\text{eff}}$ of the damage gradient function $\Delta D(t)$ may be detected either as square mean of all samples $\Delta D(t)$:

$$\Delta D_{\text{eff}}^2 = \lim_{T \to \infty} \frac{1}{T} \int_0^T \Delta D^2(t)\, \mathrm{d}t \tag{17}$$

where $T$: time area
or by means of the total power $S_{\Delta D\,\text{ges}}$:

$$\Delta D_{\text{eff}}^2 = S_{\Delta D\,\text{ges}} = \int_0^\infty \tilde{G}_{\Delta D}(f)\, \mathrm{d}f \tag{18}$$

where $\tilde{G}_{\Delta D} = 2\, \tilde{S}_{\Delta D}$: one-side power spectral density and
$f$: frequency

$\tilde{G}_{\Delta D}(f)$ arises from the complex amplitude density $\tilde{A}_{\Delta D}(jf)$:

$$\tilde{G}_{\Delta D}(f) = 2 \cdot \tilde{A}_{\Delta D}(jf) \cdot \tilde{A}_{\Delta D}^*(jf)\, \mathrm{d}f \tag{19}$$

where ( )* conjugate complex part, $j = \sqrt{-1}$
Finally, the Fourier transformation of the real function $\Delta D(t)$ supplies the amplitude density:

$$\tilde{A}_{\Delta D}(jf) = \int_{-\infty}^{\infty} \Delta D(t) \cdot \exp[-j \cdot 2\pi f t]\, \mathrm{d}t \tag{20}$$

## 4. The factor *u*

Because $\Delta D(t)$ is a sine function $\sigma(t)$ powered by $\varphi$, the $\sqrt{2}$ ratio (13) is no longer valid. The new factor u has to be calculated by:

$$\overline{\overline{\Delta D_K^2}} = u \cdot \Delta D_{\text{eff}} \tag{21}$$

and

$$\left(\frac{\overline{\Delta D_K^2}}{\Delta D_{\text{eff}}}\right)^2 = \frac{1}{u^2} = \lim_{T \to \infty} \frac{1}{T} \int_0^T \sin^{2\varphi} \omega t\, \mathrm{d}t \tag{22}$$

where $\varphi$: exponent of $s$-$N$-curve, $\omega = 2\pi f$: angular frequency
Dependent on a integer-valued exponent $\varphi$, Eq. (22) yields:

$$u^2(\varphi = n) = \frac{2}{1} \cdot \frac{4}{3} \cdot \frac{6}{5} \cdot \frac{8}{7} \cdot \ldots \cdot \left(\frac{2n}{2n-1}\right) \tag{23}$$

or for any real exponents $\varphi$:

$$u^2(\varphi) = \frac{\sqrt{\pi} \cdot \Gamma(\varphi + 1)}{\Gamma\left(\frac{\varphi}{2} + 1\right)} \tag{24}$$

The factor $u$ in Eq. (24) is exactly true for narrow-band processes. An approach was proposed [9] for the case of a broad-band process. This formula involves an additonal dependence on the irregularity factor $i$:

$$u_B(\varphi, i) = u(\varphi) \cdot \sqrt{\frac{1 + i^2}{2}} \geqq 1 \tag{25}$$

where $i = n_0/n_1$ and
$n_0$: total number of crossings of load mean in one direction
$n_1$: total number of maxima

Considering an ideal narrow-band process the irregularity factor i becomes 1 and therefore $u_B(\varphi, i)$ goes over to $u(\varphi)$.

### 5. Distribution independent fatique life of narrow-band processes

Substituting Eqs. (18) and (21) in Eq. (15) provides:

$$N_L = \frac{1}{u(\varphi) \cdot \sqrt{\int_0^\infty \tilde{G}_{\Delta D}(f)\,\mathrm{d}f}} \tag{26}$$

Eq. (26) allows us to calculate any narrow-band processes of finite total power. In accordance with the usual description of the Palmgren/Miner formula Eq. (26) may also outlined as a $k$-step formula realizing the fact that the root mean square of step gradients, see Eq. (14), also consists of a sum formula:

$$N_L = \sqrt{\frac{\sum_{K=1}^{m} n_K}{\sum_{K=1}^{m} \frac{n_K}{N_K^2}}} \tag{27}$$

Eq. (27) can be explained physically as an 'Accumulation of Abstract Damage Power' [10] similar to Palmgren/Miner's rule as an accumulation of an abstract damage energy.

### 6. Distribution independent fatique life of broad-band processes

In this case the performance spectrum of different frequencies influences simultaneously. Therefore it is better to replace 'Number of Cycles $N$' by 'Time $T$'. For an ideal narrow-band process it is true that the total power in the shape of the one-side power spectrum $G$ is only concentrated on a single frequency $f_0$:

$$S_{\Delta D_{\mathrm{ges}}} \approx G_{\Delta D}(f_0) = \Delta D_{\mathrm{eff}}^2 \tag{28}$$

Substituting Eq. (28) in Eq. (26) yields:

$$N_L = \frac{1}{u(\varphi) \cdot \sqrt{G_{\Delta D}(f_0)}} \tag{29}$$

$N_L$ may be split into 'Fatigue Life Time $T_L$ and frequency $f_0$:

$$N_L = T_L \cdot f_0 \tag{30}$$

Then Eq. (29) leads to:

$$T_L = \frac{1}{u(\varphi) \cdot \sqrt{G_{\Delta D}(f_0) \cdot f_0^2}} \tag{31}$$

The product under the root represents the power spectrum of a speed function $\Delta D_v(t) = \mathrm{d}[\Delta D(t)]/\mathrm{d}t$. Thus

$$T_L = \frac{2\pi}{u(\varphi) \cdot \sqrt{G_{\Delta D_v}(f_0)}} = \frac{2\pi}{u(\varphi) \cdot S_{\Delta D_{v\,\mathrm{ges}}}} \tag{32}$$

respectively, where $S_{\Delta D_{v\,\mathrm{ges}}}$ is the total power of speed $\Delta D_v(t)$. This power may be separated in its power spectral density parts $\tilde{G}_{\Delta D_v}(f)$ according to Eq. (18). In consideration of $\tilde{G}_{\Delta D_v}(f) = 4\pi^2 \cdot f^2 \cdot \tilde{G}_{\Delta D}(f)$ and changing $u(\varphi)$ to $u_B(\varphi, i)$ Eq. (32) yields the fatigue life time of broad-band processes:

$$T_L = \frac{1}{u_B(\varphi, i) \cdot \sqrt{\int\limits_0^\infty \tilde{G}_{\Delta D}(f) \cdot f^2 \, \mathrm{d}f}} \tag{33}$$

Eq. (33) contains Eq. (26) as special case.

## 7. Application of discrete Fourier transformation to the damage gradient function

An actual calculation of fatigue life time $T_L$ for any load time courses of finite total power is only possible on the basis of samplification. Then a discrete frequency spectrum is obtained by means of discrete Fourier transformation. In this case the infinite integral in Eq. (33) goes over to a finite sum formula until a finite number $p$ of spectral values and discrete frequencies $f_l$:

$$T_L = \frac{1}{u_B(\varphi, i) \cdot \sqrt{\sum\limits_{l=1}^{p} G_{\Delta D l}(f_l) \cdot f_l^2}} \tag{34}$$

where $G_{\Delta D_l}$: discrete power spectrum

Before being transformed into the frequency domain, the load time course has to be changed in a discrete damage gradient function, e.g. according to Eq. (16).
The use of Fast Fourier Transformation (FFT) [11] makes possible an effective calculation of fatigue life. However, the user of Eq. (34) has to consider the usual peculiarities of digital signal analysis like Shannon's scanning theorem and unavoidable leakages due to finite time lenghts.
The FORTRAN program SLEBE [9] was implemented for the calculation of the fatigue life time $T_L$ according to Eq. (34). The prerequisite for using this program is the previous recording of the sampled load time course on a magnetic disc file. SLEBE changes this discrete function to a corresponding damage gradient function, considering top stress $\sigma_{\max}$, the $s$-$N$-curve, and the endurance limit $\sigma_D$. After that operation the new values $\Delta D(t)$ are recorded on a second disc file for further processing.
As already mentioned, only values above the mean value are used. Hence, transient time parts occur, which would create additional mistakes during the convoluting operation in Fourier transformation, see Fig. 2. Therefore, the program puts together the transient parts. This treatment leads to a frequency which is too high. In Eq. (34) we must therefore consider a 'Transient Factor $t_F = T_F/T$':

$$T_L = \frac{1}{u_B(\varphi, i) \cdot t_F \cdot \sqrt{\sum\limits_{l=1}^{p} G_{\Delta D l}^*(f_l) \cdot f_l^2}} \tag{35}$$

where $G^*$: power spectrum of transient parts

The program transforms successive time blocks with a maximal block length $n = 2048$ samples and finally calculates the arithmetic mean of power spectra of all processed blocks for every frequency $f_l$.

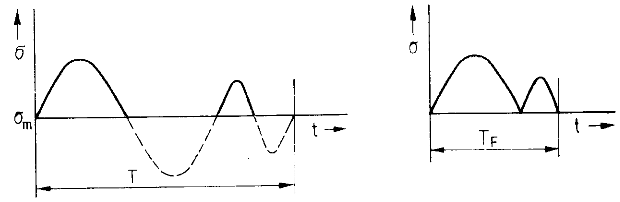


Fig. 2 Processing of values above mean stress $\sigma_m$ as a transient function

Furthermore, the function $\Delta D(t)$ may be multiplied by weighting functions in time domain. Window functions according to Bartlett, Hanning, and Hamming [12] are used.

## 8. Examples of application

If Eq. (34), or (35), is really independent of amplitude distribution, then it has to be possible to predict the fatigue life of an usual sine function, see Fig. 3, simply by means of its samples.

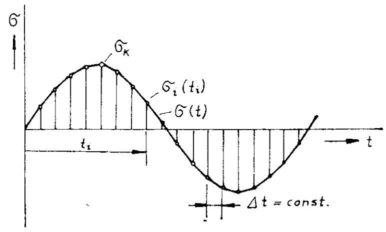


Fig. 3 Sine function $\sigma(t)$, samples $\sigma_i(t_i)$, amplitude $\sigma_k$, scan interval $\Delta t$

A sine function is given:

$$\sigma(t) = \sigma_K \cdot \sin 2\pi f_0 t \tag{36}$$

and an $s$-$N$-curve with:

$$N(\sigma) = 1.27 \cdot 10^{17} \cdot \sigma^{-5.42} \tag{37}$$

At $\sigma_K = 180$ MPa the actual fatigue life is $N_E(\sigma_K) = 75897$ cycles, and due to a frequency $f_0 = 20$ Hz the fatigue life time would be $T_E = 3794.85$ sec.
For calculation by SLEBE the function $\sigma(t)$ was digitally produced as a value sequence $\sigma_i(t_i)$ with 50 samples per complete cycle and recorded on disc. The program using different window functions provided the following results:

Bartlett: $T_L = 3897.7$ sec (0.97)
Hanning: $T_L = 3697.7$ sec (1.03)
Hamming: $T_L = 3855.6$ sec (0.98)

The figures in brackets show the accuracy $Q$, i.e. the actual fatigue life time $T_E$ divided by the calculated fatigue life time $T_L$.
This test example took about three minutes including dialogue for the whole processing of 10,000 samples $\sigma_i$ by a computer with 1 MHz CPU frequency.
For verification with experimental values, random functions $\sigma(t)$ were analyzed by SLEBE [9], which Lange [13] had detected in their effect on notched steel specimens St38. He had used Gaussian processes of various irregularity factors $i = 0.95$, $i = 0.7$, and $i = 0.3$ and with a uniform Crest factor $C = \hat{\sigma}_{max}/\sigma_{eff} = 3.7$. In Fig. 4 the load time course of $i = 0.7$ is plotted.

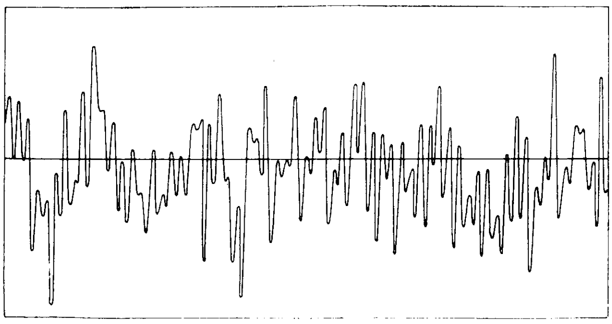

Fig. 4 Example for a used Gaussian process with $i = 0.7$

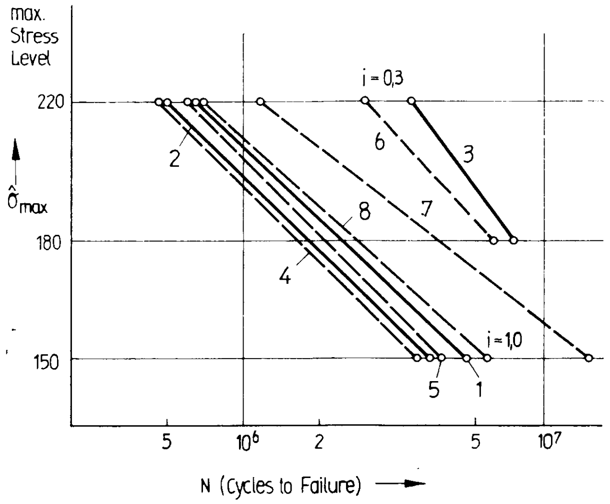


Fig. 5 Exp. fatigue life curves: (1) $i = 0.95$ (2) $i = 0.7$ (3) $i = 0.3$, calculated by SLEBE: (4) $i = 0.95$ (5) $i = 0.7$ (6) $i = 0.3$, analytically detected for $i = 1.0$: (7) according to Miles, (8) after Eq. (38)

For each of these functions, 100,000 samples were processed by SLEBE. Fig. 5 shows the experimental results by Lange in comparison to the calculated values. These results are drawn as fatigue life curves. In Fig. 5 two analytically calculated fatigue life curves (7) and (8) are shown for the theoretical irregularity factor $i = 1$. Curve (7) is based on Eq. (3) and curve (8) was derived on the basis of the distribution independent fatigue life prediction specially applied to a narrow-band Gaussian process. Similar to Miles, the sums in Eq. (27) were replaced by infinite integrals of the Rayleigh density and the Gamma function was again used for effective description of these integrals. Hence, $N_L$ goes over to a fatigue life $N_{LG}$ particular to the case of a narrow-band Gaussian process:

$$N_{LG} = \frac{K_w}{\sqrt{\Gamma(\varphi + 1)} \cdot 2^{\frac{\varphi}{2}} \cdot \sigma_{\text{eff}}^{\varphi}} \tag{38}$$

**9. Survey of the proposed fatique life theory**

To get an impression of the utilization of the proposed theory, the above mentioned accuracy $Q = T_E/T_L$ was applied. In total, 34 test series of random and non-random load time functions were analyzed by the SLEBE program [9]. The accuracy in comparison to the experimental fatigue supplied logarithmic normally distributed results with the following values: mean = 1.26, medien = 1.15. 10. p.c. of accuracy was smaller than $Q = 0.39$ and 90.p.c. was smaller than 2.43. Such scatter is usual for fatigue problems.

The following steps have to be performed in the spectroanalytical calculation:

1. Samplification of the load time function $\sigma(t)$ to $\sigma_i(t_i)$
2. Division of $\sigma_i(t_i)$ by its constant mean stress $\sigma_m$.
   Detecting the irregularity factor i and the absolute maximum $\hat{\sigma}_{\max}$ of the $\sigma_i(t_i)$ function (with zero mean)
3. Changing function $\sigma_i(t_i)$ to a damage gradient function $\Delta D_i(t_i)$, e.g.:

$$\Delta D_i(t_i) = \frac{1}{K_w} \cdot \sigma_i^{+\varphi} \qquad (39)$$

   where $K_w$ and $\varphi$: $s$-$N$-curve constants considering the constant mean stress $\sigma_m$.
4. Calculation of discrete power spectrum $G_{\Delta D_l}$ by means of Fast Fourier Transformation.
5. Application of Eq. (34) or (35).

For a manual calculation Eq. (27) is suitable. However, it is only valid for exact or approximate narrow-band processes.

## 10. Conclusions

In this paper is shown that additional information on distribution functions is always necessary when the linear cumulative damage hypothesis by Palmgren Miner is used, because this formula represents an arithmetic mean. Such a mean is not sufficient as a sole basis for spectro-analysis. Therefore a square mean is proposed. This change of the cumulative damage hypothesis enables us to use samples without reference to amplitude distributions.

The calculation of the fatigue life time $T$ on the basis of samples was here derived and demonstrated for spectro-analysis. However, there is still a further facility to obtain this life time. The right term of Eq. (32) contains the total power of the speed time function $\Delta D_V(t)$, i.e. the root mean square $\Delta D_{V\text{eff}}$ of speed values $\Delta D_V(t)$. $\Delta D_{V\text{eff}}$ may be detected as sum of all squared speed samples according to Eq. (17). The speed gradients $\Delta D_V$ are calculable by means of the values $\Delta D$, for example on basis of Spline functions. On this way the fatigue life time $T_L$ can be calculated without Fourier transformation.

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